SET THEORY

To the concept of set can be very easy to get empirical way, looking different groups, many kind of objects, things, and other living creatures. So we have set inhabitants of a city, a set of books in the library, a set of benches in the classroom, etc.. Maker of set theory is **Georg Kantor,** german mathematician, who was first given "descriptive" definition of sets. Many other mathematicians are also trying to define set. **Today, in modern understanding, the concept of sets is not defined, but adopts intuitive as a whole some diferent facilities.** Cases which is a set composed are called **elements of** set. There are a lot of sets with the final elements, which we call **the final sets,** and sets with infinite many elements, **infinite sets.**

So, for example, set all the inhabitants of the earth is one final set, while a set of all whole numbers contains infinite many elements.

Set we celebrated with large letters A, B, ,..... X, Y, ... , and elements of sets with small letters a,b …,x,y…

If x is an element of X, that fact will show with $x \in X$, and if x do not belong to set X, $x \notin X$. This label $x \notin X$, we will read "x does not belong to a set of X"

Place now the question: "How many elements have a set of natural numbers, larger than one and smaller of two? It is clear that there is no such set of elements. For such a set we say that it is **empty** and marked with $\varnothing$. Empty set is a subset of each set.

However, sometimes is not useful, and not possible to directly mention all elements of a set. Therefore, it is used, and this recording of sets: $\{x \ : S(x)\}$. For example set $X=\{7,8,9,10,11,12\}$ we can write as follows: $X=\{x \ : \ x \in N \wedge 6< x <13 \}$.

Two sets are **same** if all the elements of a first set , at the same time , are elements of a second set, and vice versa. **Notes: A=B if and only if** $(\forall x)(x\in A \Leftrightarrow x \in B)$ **.**

For example, by definition, will be (a, a, a, b, b, c) = (a, b, b, c, c, c) = (a, b, c).

So, each member of the set is present with one occurrence, and all the rest of his impressions, if any, are not important.

We say that a set of B is **subset** of A, B$\subset$A, if all the elements of set B are also

elements of A. **B$\subset$ A if and only if ($\forall$ x) (x $\in$B $\Rightarrow$ x$\in$ A)**

Relation introduced by this definition is called **relation of inclusion .**

Operations with sets

- ***UNION***
- ***INTERSECTION***
- ***DIFFERENCE***
- ***SYMMETRIC DIFFERENCE***
- ***PARTITIVE SET***
- ***CARTESIAN PRODUCT***
- ***COMPLEMENT OF SET***

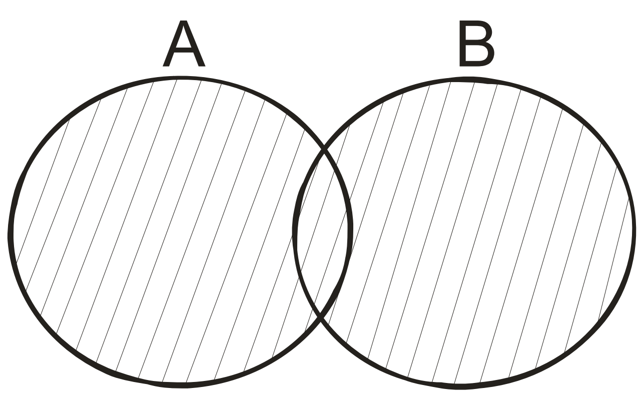
UNION

Set of all the elements who are elements at least one of the set A **OR** set B, we called th

the Union of sets A and B, and marked with: A$\cup$B.

$A \cup B = \{x | x \in A \vee x \in B\}$

On the chart would look like this:



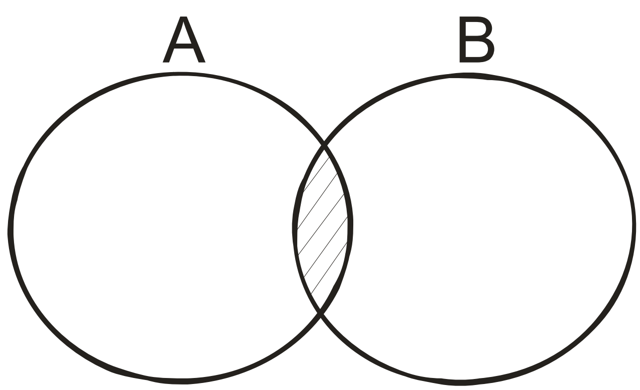
Example: If A={1,2,3} and B={2,3,4} then A$\cup$B={1,2,3,4}

INTERSECTION

Set of all the elements who are elements of set A **and** set B we called the intersection A and B, and marked with $A \cap B$.

$A \cap B = \{x | x \in A \wedge x \in B\}$

Graphic view would be:



Example: If A={1,2,3} and B={2,3,4} then $A \cap B$={2,3}

DIFFERENCE

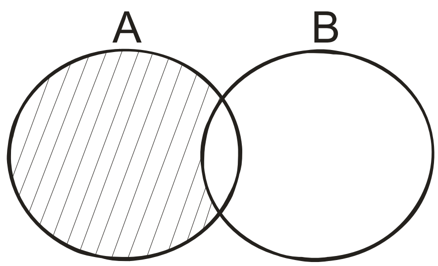
Set of all the elements who **are elements of A and are not elements of B** we called the difference, and marked with A\B.

$\mathbf{A \setminus B = \{ x : x \in A \wedge x \notin B \}}$

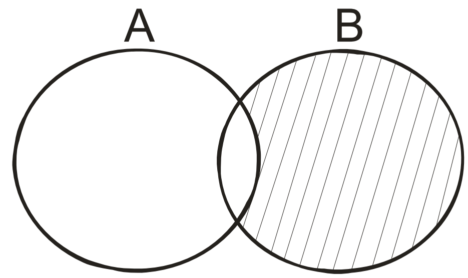
Of course, we can observe and set B \ A, it would be all the elements of B who are not in A.

$\mathbf{B \setminus A = \{ x : x \in B \wedge x \notin A \}}$

On the diagrams would look like this:



A\B



B\A

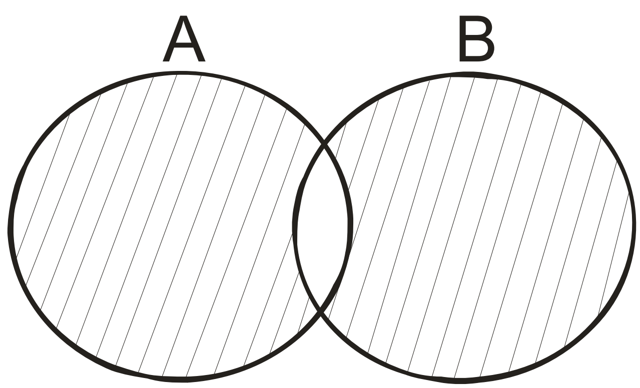
Example: If $A=\{1,2,3\}$ and $B=\{2,3,4\}$ then $A\setminus B = \{1\}$ and $B\setminus A= \{4\}$

SYMMETRIC DIFFERENCE

Set $(A\setminus B)\cup(B\setminus A)$ is called a symmetrical difference and is usually celebrated with Δ.

$\mathbf{A\,\Delta\, B= (A\setminus B)\cup(B\setminus A).}$

On the diagrams would look like this:



Example: If $A=\{1,2,3\}$ and $B=\{2,3,4\}$ then $\mathbf{A\,\Delta\, B=\{1,4\}}$

PARTITIVE SET

Set of all subsets of A is called **partitive set** of A and marked with P(A).

Example:

If $A=\{1,2,3)$, then $P(A)=\{\ \varnothing, \{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\}, \{1,2,3\}\}$

CARTESIAN PRODUCT

The famous French philosopher and mathematician Dekart in Mathematics introduced the concept of coordinate system, which is today in his honor, called Cartesian system. In this system, each pair of numbers (x, y) corresponds exactly one point in plane , and vice versa.

(x,y)=(a,b) if and only if x=a $\wedge$ y=b

Cartesian product is a set:

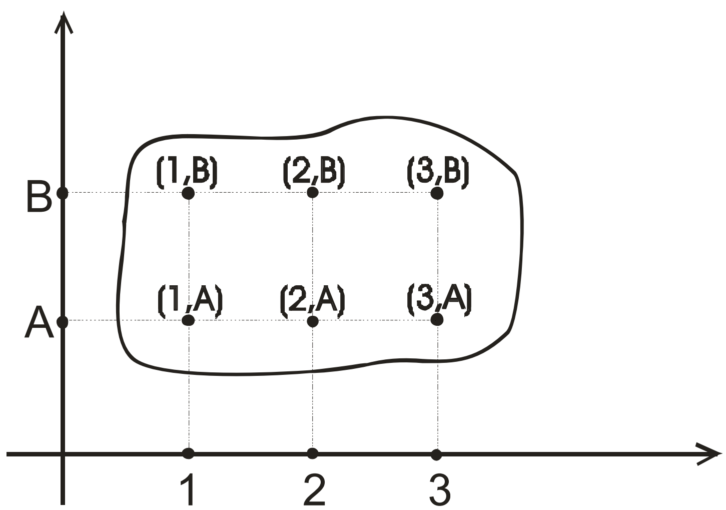
$$A \times B = \{(a,b) \mid a \in A, b \in B\}.$$

$\mathbf{A} \times B \neq B \times \mathbf{A}$

Example:

If M={1,2,3} and N={A,B} then: M×N={(1,A),(1,B),(2,A),(2,B),(3,A),(3,B)}

In the picture:

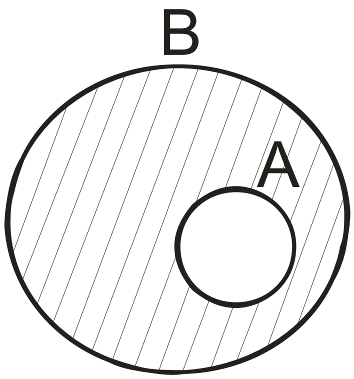


COMPLEMENT OF SET

Union, intersection and symmetric difference are binary set operations, while the complement of set is unary operation. This is a collection of all the elements are not included in the set observed.

Complement is marked with $\overline{A}$.

The picture would be:



$\overline{A}$ **={x: x$\notin$ A}**

Example: If A={1,3,7} and B={1,2,3,4,5,6,7} then: $\overline{A} = \{2,4,5,6\}$

EXAMPLES:

1) **Prove that the empty set is a subset of each set.**

Proof:

We actually need to prove that: $\varnothing \subset A \Leftrightarrow (\forall x)(x \in \varnothing \Rightarrow x \in A)$

How empty set has no elements, the value of $x \in \varnothing$ is certainly incorrect.

So: $\perp \Rightarrow x \in A$

p	q	p⇒q
T	T	T
T	⊥	⊥
⊥	T	T
⊥	⊥	T

'from incorrect follows all' (always correct)

Therefore, the empty set is a subset of each set.

2) **Sets are : A={x : x contained in 12, $x \in N$}**
B={x : x contained in 20, $x\in N$}
C={x : x contained in 32, $x \in N$}.

Find sets: $A\backslash(B\cup C)$, $A\cup(B\cap C)$, i $A\backslash(B\backslash C)$

First, we must determine the sets: A, B and C.

As the number 12 can be shared with 1,2,3,4,6,12, we have:

A={1,2,3,4,6,12} and similar:
B={1,2,4,5,10,20}
C={1,2,4,8,16,32}

$A\backslash(B\cup C)$=?

$B\cup C$={1,2,4,5,8,10,16,20,32}
We now require those who are in set A and are not in set $B\cup C$.
$A\backslash(B\cup C)$={3,6,12}

$A\cup(B\cap C)$=?

$B\cap C$={1,2,4}
$A\cup(B\cap C)$={1,2,3,4,6,12}.

$A\backslash(B\backslash C)$=?

$$\begin{aligned} A\backslash(B\backslash C) &= \{1,2,3,4,6,12\}\backslash(\{1,2,4,5,10,20\}\backslash\{1,2,4,8,16,32\}) \\ &= \{1,2,3,4,6,12\}\backslash\{5,10,20\} \\ &= \{1,2,3,4,6,12\} \end{aligned}$$

3) **Sets are A={1,2,3,4,5} and B={4,5,6,7}. Define a set X to be:**

$X\backslash B=\emptyset$ i $A\backslash X$ ={1,2,3}

The solution**:**

It seems that we here have more opportunities for requested set X.

How is $X\backslash B=\emptyset$, it tells us that all the elements of B are potential elements of X because there are no such elements are in X and not in a B.
$A\backslash X$ ={1,2,3} tells us that in a set X are not elements 1,2,3.

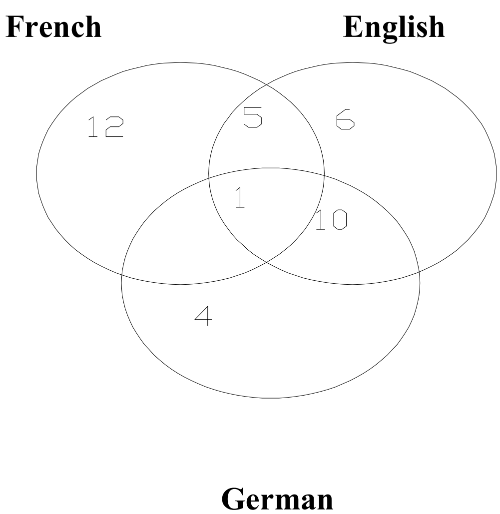
So: X={4,5} or X={4,5,6}or X={4,5,7}or X={4,5,6,7}

4) On one foreign language course each listener learns at least one of the three languages (English, French or German) and: 18 people learning French, 22 learning English, 15 listeners learn German, 6 listeners learn English and French, 11 English and German, 1 listener learns all three languages.
How many listeners are on the course and how many of them learning only two languages?

The solution: First, records inspected data:

- 18 people learning French
- 22 learn English
- 15 people learning German
- 6 people learn English and French
- 11 people in English and German
- 1 listener learn all three languages

It is best to use diagram with three sets:



First entries 1 in intersection of all three sets.
Then, intersection of French and English, we not writing 6 ,but 6-1 =5
Intersection English and German 11-1 = 10
Next , 18-5-1 = 12 that learn only French
22-10-5-1 = 6 to learn English
Finally, 15-10-1 = 4 to learn German.

The number of listeners is 12 +5 +6 +1 +10 +4 = 38,
and the number of those who learn only two languages 10 + 5 = 15

5) **Prove set equality:** $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

Proof:

$(\forall x)\ (x \in (A \cup B) \cap C) \Leftrightarrow (x \in (A \cap C) \cup (B \cap C))$

$(x \in (A \cup B) \wedge x \in C) \Leftrightarrow (x \in (A \cap C) \vee x \in (B \cap C))$

$((x \in A \vee x \in B) \wedge x \in C) \Leftrightarrow ((x \in A \wedge x \in C) \vee (x \in B \wedge x \in C))$

we take: $p = x \in A$
$q = x \in B$
$r = x \in C$

We have received a formula: **F: $((p \vee q) \wedge r) \Leftrightarrow ((p \wedge r) \vee (q \wedge r))$**

We must prove formula through the use of logical tables and operations:

p	q	r	$p \vee q$	$(p \vee q) \wedge r$	$p \wedge r$	$q \wedge r$	$(p \wedge r) \vee (q \wedge r)$	F
T	T	T	T	T	T	T	T	T
T	T	$\perp$	T	$\perp$	$\perp$	$\perp$	$\perp$	T
T	$\perp$	T	T	T	T	$\perp$	T	T
T	$\perp$	$\perp$	T	$\perp$	$\perp$	$\perp$	$\perp$	T
$\perp$	T	T	T	T	$\perp$	T	T	T
$\perp$	T	$\perp$	T	$\perp$	$\perp$	$\perp$	$\perp$	T
$\perp$	$\perp$	T	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	T
$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	T

$\Leftrightarrow$

Formula is tautology, and thus completed the proof.

6) **Prove set equality:** $C\backslash(A\cap B)=(C\backslash A)\cup(C\backslash B)$

Proof:

$$(\forall x)(x\in C\backslash(A\cap B)) \Leftrightarrow (x\in(C\backslash A)\cup(C\backslash B))$$
$$(x\in C \wedge x\notin(A\cap B)) \Leftrightarrow (x\in(C\backslash A) \vee x\in(C\backslash B))$$
$$(x\in C \wedge \neg(x\in A\wedge x\in B)) \Leftrightarrow (x\in C\wedge\neg(x\in A)) \vee (x\in C\wedge\neg(x\in B))$$

$p= x\in A$
$q= x\in B$
$r = x\in C$

F: $\mathbf{(r\wedge\neg(p\wedge q)) \Leftrightarrow ((r\wedge\neg p) \vee (r\wedge\neg q))}$

p	q	r	$\neg p$	$\neg q$	$p\wedge q$	$\neg(p\wedge q)$	$r\wedge\neg(p\wedge q)$	$r\wedge\neg p$	$r\wedge\neg q$	$(r\wedge\neg p)\vee(r\wedge\neg q)$	F
T	T	T	$\perp$	$\perp$	T	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	T
T	T	$\perp$	$\perp$	$\perp$	T	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	T
T	$\perp$	T	$\perp$	T	$\perp$	T	T	$\perp$	T	T	T
T	$\perp$	$\perp$	$\perp$	T	$\perp$	T	$\perp$	$\perp$	$\perp$	$\perp$	T
$\perp$	T	T	T	$\perp$	$\perp$	T	T	T	$\perp$	T	T
$\perp$	T	$\perp$	T	$\perp$	$\perp$	T	$\perp$	$\perp$	$\perp$	$\perp$	T
$\perp$	$\perp$	T	T	T	$\perp$	T	T	T	T	T	T
$\perp$	$\perp$	$\perp$	T	T	$\perp$	T	$\perp$	$\perp$	$\perp$	$\perp$	T

$\Leftrightarrow$

So this formula is tautology, and the initial set equality **is correct**